1212-17-505-011

Code No. 7404 / CORE

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, Dec. 2018 / Jan. 2019

Subject: Mathematics

Paper – I

Complex Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B. Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Find all roots of cos z = 2.

2 Find $|\cosh z|^2$

3 Show that if C is a positively oriented simple closed contour, then the area of the region enclosed by C is $\frac{1}{2i}\int_C \bar{z}\, dz$.

4 State and prove the principle of deformation of paths.

5 Show that $\sum_{n=1}^{\infty} r^n \cos n\theta = \frac{r\cos\theta - r^2}{1 - 2r\cos\theta + r^2}$, where $0 < r < 1$.

6 Find the Laurent series of $f(z) = \frac{1}{z^2(1-z)}$, $1 < |z| < \infty$.

7 Compute the linear fractional transformation that maps 2, i, -2 onto 1, i, -1.

8 Find the fixed points of $w = \frac{z-1}{z+1}$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Find the harmonic conjugates of
 i) u(x,y) = sinhx siny
 ii) $u(x,y) = 2x - x^3 + 3xy^2$.

OR

b) i) State and prove the reflection principle.
 ii) Prove that, a function that is analytic in a domain D is uniquely determined by its values in a domain or along a line segment contained in D.

10 a) i) Prove Cauchy's inequality.
 ii) Prove Liouville's theorem.

OR

b) i) Prove maximum modulus principle.
 ii) Find the absolute maximum of f(z) = sin z over the region $[0, \pi] \times [0, 1]$.

11 a) State and prove Laurent's theorem.

OR

b) Prove that $\operatorname{Res}_{z=\infty} f(z) = -\operatorname{Res}_{z=0}\left(\frac{1}{z^2} f\left(\frac{1}{z}\right)\right)$ and hence evaluate $\int_{|z|=2} \frac{5z-2}{z(z-1)} dz$, |z|=2 is positively oriented.

12 a) Evaluate $\int_0^\infty \frac{x^2}{1+x^6} dx$.

OR

b) Evaluate P.V. $\int_{-\infty}^{\infty} \frac{(x+1)\cos x}{x^2+4x+5}$ dx.

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1212-17-505-011)

Code No. 7406 / CORE

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, Dec. 2018 / Jan. 2019

Subject: Mathematics

Paper – II

Functional Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Let E be a complete normed linear space over IR or ¢. If $\{x_n\}, \{y_n\} \subset E_n, \alpha_n \in$ IR or ¢ and $x_n \to x$, $y_n \to y$ respectively as $n \to \infty$ and $\alpha_n \to \alpha$ as $n \to \infty$ then prove that
 i) $x_n + y_n \to x+y$
 ii) $\alpha_n x_n \to \alpha x$ as $n \to \infty$

2 Prove that on a finite dimensional normed linear space any norm $||.||$ is equivalent to any norm $||.||'$.

3 State that prove parallelogram law.

4 Prove that the space ℓ_p with $p \neq 2$ is not a Hilbert space.

5 Prove that every linear functional is homogeneous.

6 Let E_x be a normed linear space over IR or ¢. If A, B ϵ $(E_x \to E_x)$ then prove that $AB \epsilon (E_x \to E_x)$ and $||AB|| \leq ||A||\ ||B||$.

7 Define projection operator. Also prove that if P is projection then I-P is projection.

8 Define:
 i) Normal operator
 ii) Unitary operator.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Let $\{e, e_2, \ldots, e_n\}$ be a linearly independent set of vectors in a normed linear space E (of any dimension) then there is a number C > 0 such that for any choice of scalars $\alpha_1, \alpha_2, \ldots, \alpha_n$, we have
 $||\alpha_1 e_1 + \alpha_2 e_2 + \ldots + \alpha_n e_n|| \geq c(|\alpha_1| + |\alpha_2| + \ldots + |\alpha_n|)$

OR

 b) Let Y and Z be subspaces of a normed linear space X and suppose that Y is closed and is a proper subset of Z then prove that for every $\theta \in (0,1)$ there is a $z \epsilon Z$ such that $||z|| = 1$, $||z-y|| \geq \theta$ for all $y \in Y$.

..2

10 a) Let H be a Hilbert space. If $x \in H$ and L is some closed subspace of H then prove that x has a unique representation of the form $x = y + z$ with $y \in L$ and $z \perp L$.

OR

b) Let H be a Hilbert space and L is closed, convex set in H and $x \in H-L$, then there is a unique $y_o \in L$ such that $\|x - y_o\| = \inf_{y \in L}\{\|x-y\|\}$.

11 a) Prove that $\{A_n\}$ converges uniformly if and only if $\{A_n\}$ converges uniformly for every $x \in E_x$.

OR

b) State and approve generalized Hahn-Banach theorem.

12 a) State and prove closed graph theorem.

OR

b) Prove that every self adjoint operator A generates some bounded bilinear hermitian form $A(x,y) = \langle Ax,y \rangle = \langle x,Ay \rangle$. Conversely if a bounded linear Hermitian form A(x,y) is given, then it generates some self adjoint operator A, satisfying the equality $A(x,y) = \langle Ax,y \rangle$

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1212-17-505-011

Code No. 7408 / CORE

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, Dec. 2018 / Jan. 2019

Subject: Mathematics

Paper – III : (Elective – A)
Discrete Mathematics

Time: 3 Hours Max.Marks: 80

**Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in part-A and 12 marks in Part-B.**

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 In a lattice (L, ≤) for a, b, c ∈ L prove that $b \le c \Rightarrow \begin{cases} a * b \le a * c \\ a \oplus b \le a \oplus c \end{cases}$

2 Define
i) Lattice homomorphism
ii) Direct product of lattices

3 In a Boolean algebra show that $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

4 Design a logic gate for the Boolean expression $f = (x+y)\,(\overline{xy})$.

5 Define:
i) Directed graph
ii) Weighted graph
iii) Multigraph
iv) Simple graph

6 Define Eularian and Hamiltonian paths and distinguish between them.

7 Show that a circuit and the complement of any spanning tree must have at least one edge in common.

8 Define
i) Sub graph
ii) Cut set

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Let (L, ≤) be a lattice. For any a, b, c ∈ L prove that the following distribution inequalities hold:

$$a \oplus (b * c) \le (a \oplus b) * (a \oplus b)$$

$$a * (b \oplus c) \ge (a * b) \oplus (a * c)$$

OR

b) Show that in a lattice if $a \le b \le c$ then

$a \oplus b = b * c$ and $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$.

..2

10 a) Use K-map to minimize the Boolean function f (a, b, c, d) = $\Sigma(0, 5, 7, 8, 12, 14)$

OR

b) Show that in a Boolean algebra the following expressions are equivalent to one another

i) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$

ii) $(x * z) \oplus (x' * y) \oplus (y * z)$

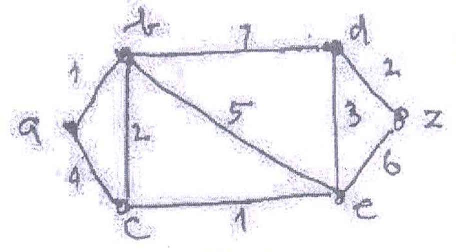
iii) $(x \oplus y) * (x' \oplus z)$

iv) $(x * z) \oplus (x' * y)$

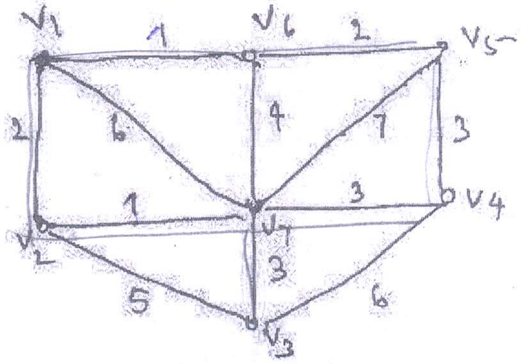
11 a) State and prove Euler's formula on planar graphs.

OR

b) Use Dijkstra algorithm to find shortest path from a to z in the following graph.



12 a) Explain Kruskal's algorithm and find a minimal spanning tree of the following graph:



OR

b) Define:

i) Binary tree

ii) Binary search tree

Show that every circuit has an even number of edges in common with every cut set.

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1212-17-505-011

Code No. 7412 / CORE

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, Dec. 2018 / Jan. 2019

Subject: Mathematics / Applied Mathematics

Paper – IV

(A) Operations Research

Time: 3 Hours Max.Marks: 80

Note: Answer all questions from Part-A and Part-B. Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Explain the graphical procedure to solve a LPP.

2 Define primal and dual problems.

3 Explain the mathematical formulation of assignment problem.

4 Write an algorithm for obtaining IBFS of transportation problem using Vogel's approximation method.

5 What is "Bellomans principle of optimality" in DPP?

6 What is dynamic programming problem? Explain.

7 Define network and explain the characteristics of the CPM.

8 Define critical path and the three time estimates in PERT.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Use Big – M method to solve the following LPP and also give its algorithm.

Minimize $Z = 600x_1 + 500x_2$

STC $2x_1 + x_2 \geq 80$

$x_1 + 2x_2 \geq 60$

$x_1, x_2 \geq 0$

OR

b) Solve the following LPP by simplex method and also give its algorithm.

Maximize $Z = 3x + 5x_2 + 4x_3$

STC $2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0$

10 a) Explain the step by step procedure to solve a transportation problem using North-West corner procedure for IBFS and MODI method for OBFS.

OR

b) A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix.

		Employees				
		I	II	III	IV	V
Jobs	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee so as to minimize the total man hours? Also write the algorithm for solving assignment problem using Hungarian method.

11 a) Solve the following using Dynamic programming technique

Maximize $Z = x_1^2 + x_2^2 + x_3^2$

STC $x_1 \cdot x_2 \cdot x_3 = 6$

$x_1, x_2, x_3 \geq 0$

OR

b) Solve the following using Dynamic programming technique

Maximize $Z = x_1 \cdot x_2 \cdot x_3$

STC $x_1 + x_2 + x_3 = 10$ and

$x_1, x_2, x_3 \geq 0$

12 a) Explain about:

i) Various floats involved in a network

ii) Forward pass method and

iii) Backward pass method.

OR

b) Draw the network diagram and obtain critical path to the following activities. Also obtain total float, free flat and independent floats of the activities.

Activity	1-2	2-3	2-4	2-5	3-5	4-5	5-6	6-7	6-8	7-8
Time (hrs)	3	3	7	9	5	0	6	4	13	10

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